جامعة بنها

كلية الحاسبات والذكاء الإصطناعي

Probability and Statistics

Lecture 03

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Faculty of Computers and Artificial Intelligence
Benha University

Spring 2023

مواعيد أول 8 محاضرات

مواعيد أول 8 محاضرات لمادة (الاحتمالات و الإحصاء)					
م	اليوم	التاريخ	المكان	الوقت	ملاحظات
1	الأربعاء	22 فبراير 2023	مدرج 3	11:15 ص	تم دراسة المحاضرة في موعدها
2	الأربعاء	01 مارس 2023	مدرج 3	11:15 ص	تم دراسة المحاضرة في موعدها
3	الأربعاء	08 مارس 2023	مدرج 3	11:15 ص	تم دراسة المحاضرة في موعدها
4	السبت	11 مارس 2023	أونلاين	5:00 م	بدلا من محاضرة (الأربعاء 15 فبراير 2023)
5	الجمعة	17 مارس 2023	أونلاين	5:00 م	بدلا من محاضرة (الأربعاء 15 مارس 2023)
6	الأربعاء	22 مارس 2023	مدرج 3	10:30 ص	Quiz (1) (يشمل أول ثلاث محاضرات)
7	السبت	25 مارس 2023	أونلاين	2:00 م	بدلا من محاضرة (الأربعاء 29 مارس 2023)
8	الأربعاء	05 ابريل 2023	مدرج 3	10:30 ص	
...	...	...	...	...	
...	...	...	...	...	

Chapter 1: Probability

- Sample Space.
- Events.
- Counting Techniques.
- Probability of an Event.
- Additive Rules.
- Conditional Probability.
- Independence, and the Product Rule.
- Bayes' Rule.

Probability of an Event

Axioms of Probability:

S Is a sample space, A is an event

$A \subseteq S$

$$\boldsymbol{P(S) = 1}$$

$$\boldsymbol{P(\emptyset) = 0}$$

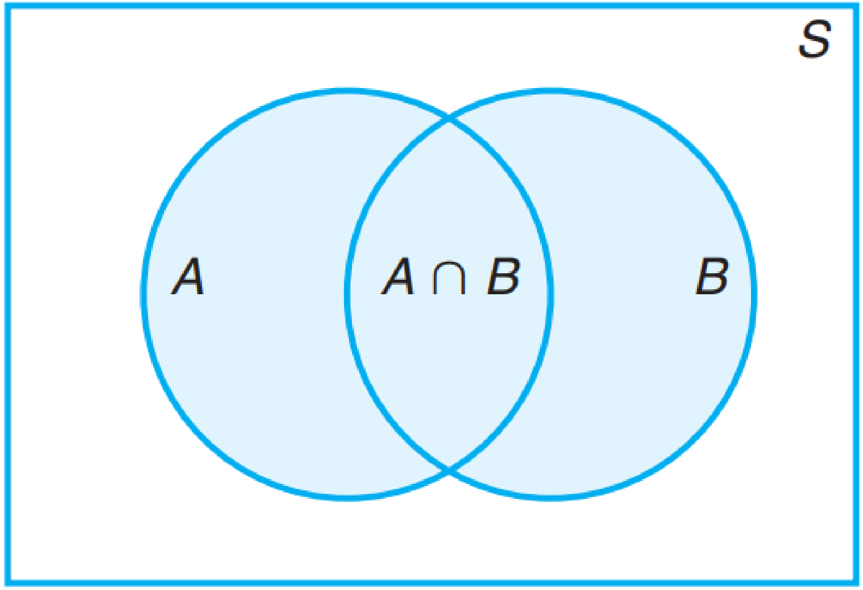
$$\boldsymbol{0 \leq P(A) \leq 1}$$

$$\boldsymbol{P(A') = 1 - P(A)}$$

Additive Rules (1/13)

If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$



Additive Rules (2/13)

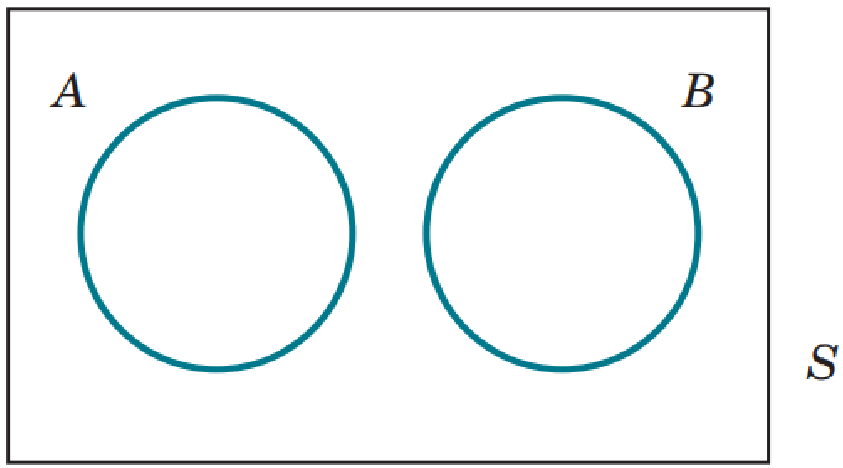
For three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Additive Rules (3/13)

If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B).$$



Additive Rules (4/13)

If $A_1,\ A_2,\ldots,A_n$ are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n).$$

Additive Rules (5/13)

Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

1) $P(A')$
2) $P(A \cup B)$
3) $P(A' \cap B)$
4) $P[(A \cup B)']$

Additive Rules (6/13)

Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

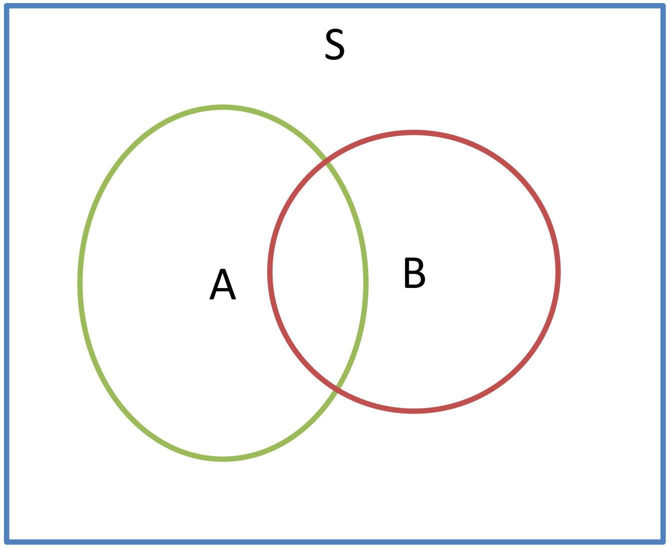
1) $P(A') = 1 - P(A) = 1 - 0.3 = 0.7$
2) $P(A \cup B) = 0.3 + 0.2 - 0.1 = 0.4$

Additive Rules (7/13)

Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

3) $P(A' \cap B)$



Additive Rules (7/13)

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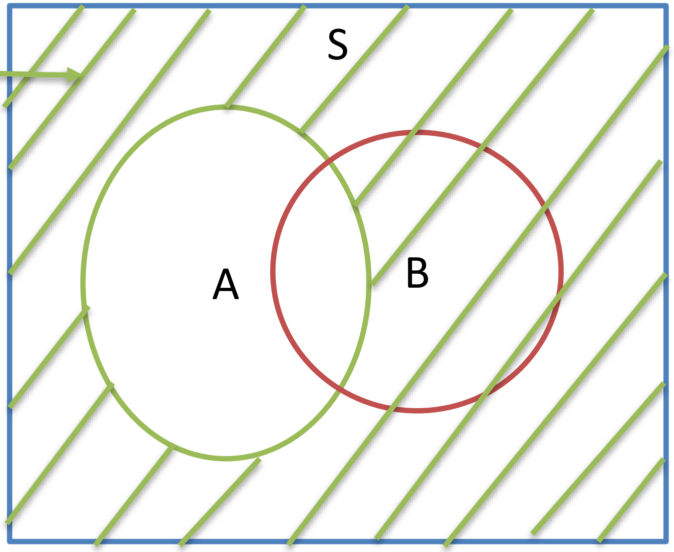

Additive Rules (7/13)

Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

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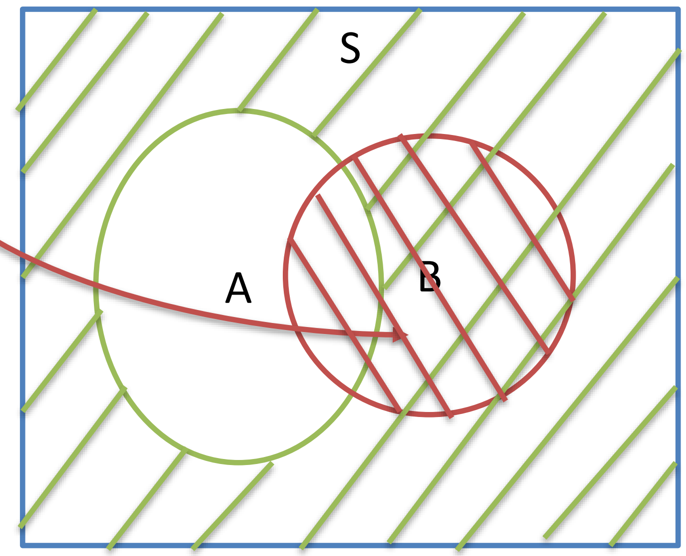


Additive Rules (7/13)

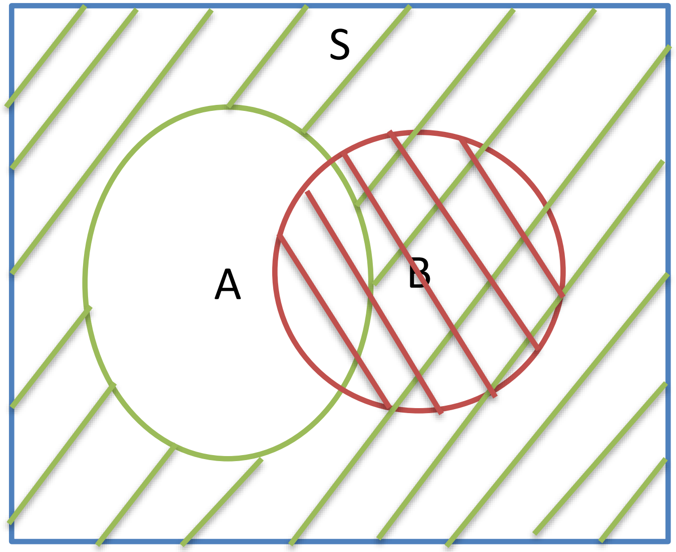
Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

3) $P(A' \cap B)$

$= P(B) - P(A \cap B) = 0.2 - 0.1$

$= 0.1$



Additive Rules (8/13)

Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

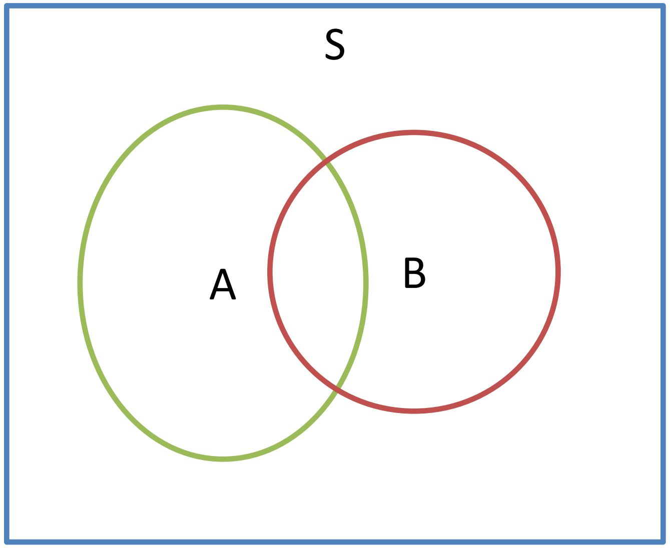
4) $P[(A \cup B)']$

Additive Rules (8/13)

Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

4) $P[(A \cup B)'] = P(A' \cap B')$



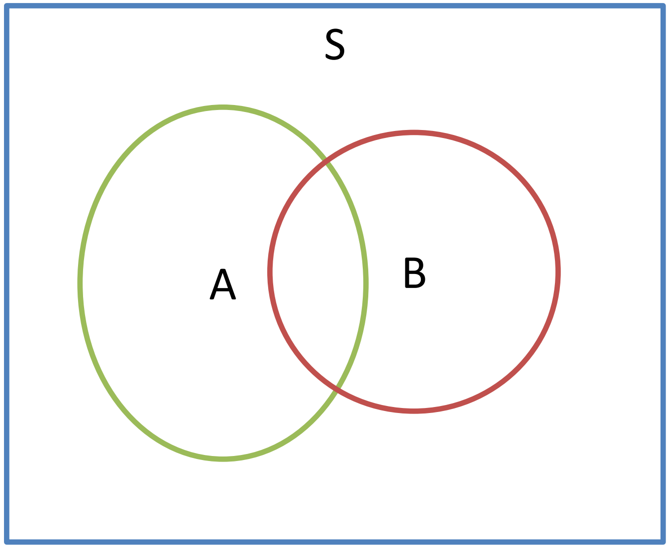
Additive Rules (8/13)

Example4:

If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$ determine the following probabilities:

4) $P[(A \cup B)'] = P(A' \cap B')$

$= 1 - P(A \cup B) = 1 - 0.4 = 0.6$



Additive Rules (9/13)

Example5:

What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?

Additive Rules (10/13)

Example5:

What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?

Solution:

Let A be the event that 7 occurs and B the event that 11 comes up.

$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26,$
$31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51,$
$52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$

Additive Rules (11/13)

Example5:

What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?

Solution:

Let A be the event that 7 occurs and B the event that 11 comes up.

$$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$$

$$P(A) = \frac{6}{36}$$

Additive Rules (12/13)

Example5:

What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?

Solution:

Let A be the event that 7 occurs and B the event that 11 comes up.

$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$

$$P(B) = \frac{2}{36}$$

Additive Rules (13/13)

Example5:

What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?

Solution:

The probability of getting a total of 7 or 11 $= P(A \cup B)$

The events A and B are mutually exclusive, since a total of 7 and 11 cannot both occur on the same toss. Therefore,

$$P(A \cup B) = P(A) + P(B) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$$

Conditional Probability (1/31)

Introduction (1/2):

Sometimes probabilities need to be reevaluated as additional information becomes available. The probability of an event B under the knowledge that the outcome will be in event A is denoted as

$$P(B \mid A)$$

and this is called the conditional probability of B given A.

Conditional Probability (1/31)

Introduction (2/2):

The conditional probability of B, given A, denoted by $P(B \mid A)$, is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad , \qquad \text{provided} \quad P(A) > 0$$

Conditional Probability (1/31)

Introduction (2/2):

The conditional probability of B, given A, denoted by $P(B \mid A)$, is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad , \qquad \text{provided} \quad P(A) > 0$$

Equally Likely Outcomes

$$\frac{P(A \cap B)}{P(A)} = \frac{\text{number of outcomes in } A \cap B}{\text{number of outcomes in } A}$$

Conditional Probability (2/31)

Example1:

As an additional illustration, suppose that our sample space *S* is the population of adults in a small town who have completed the requirements for a college degree.
We shall categorize them according to gender and employment status. The data are given in the following.

Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Conditional Probability (3/31)

Example1:

One of these individuals is to be selected at random. What is the probability that he is a male and given that he is employed?

Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Conditional Probability (3/31)

Example1:

One of these individuals is to be selected at random. What is the probability that he is a male and given that he is employed?

$$P(M|E)$$

Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Conditional Probability (3/31)

Example1:

One of these individuals is to be selected at random. What is the probability that he is a male and given that he is employed?

$$P(M|E) = \frac{P(M \cap E)}{P(E)}$$

Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Conditional Probability (4/31)

Example1:

One of these individuals is to be selected at random. What is the probability that he is a male and given that he is employed?

$$P(M \cap E) = \frac{460}{900}$$

Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Conditional Probability (5/31)

Example1:

One of these individuals is to be selected at random. What is the probability that he is a male and given that he is employed?

$$P(E) = \frac{600}{900}$$

Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Conditional Probability (5/31)

Example1:

One of these individuals is to be selected at random. What is the probability that he is a male and given that he is employed?

$$P(M|E) = \frac{460/900}{600/900} = \frac{460}{600}$$

Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Conditional Probability (6/31)

Example2:

Consider an industrial process in the textile industry in which strips of a particular type of cloth are being produced. These strips can be defective in two ways, length and nature of texture. It is known from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests. If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective?

Conditional Probability (7/31)

Example2:

Consider an industrial process in the textile industry in which strips of a particular type of cloth are being produced. These strips can be defective in two ways, length and nature of texture. It is known from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests. If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective?

$P(L) = 0.1$

Conditional Probability (7/31)

Example2:

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$P(L) = 0.1$

$P(T) = 0.05$

Conditional Probability (7/31)

Example2:

Consider an industrial process in the textile industry in which st[illegible] of a particular type of cloth are being pr[illegible]. These strip[illegible] fective in [illegible] ngth and nature of [illegible] ure. It is [illegible] from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests. If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective?

$P(L) = 0.1$

$P(T) = 0.05$

$P(L \cap T) = 0.008$

Conditional Probability (7/31)

Example2:

Consider an industrial process in the textile industry in which str[illegible] of a particular type of cloth are being pr[illegible]. These strip[illegible] fective in t[illegible] ngth and nature of [illegible] re. It is [illegible] from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests.

$P(L) = 0.1$

$P(T) = 0.05$

$P(L \cap T) = 0.008$

If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective?

$$P(T|L) = 0.008/0.1 = 0.08$$

Conditional Probability (8/31)

Example3:

A dice is rolled twice. What is the probability that the sum equal 10, if you know that the 1^{st} element equal 6?

Conditional Probability (9/31)

Example3:

A dice is rolled twice. What is the probability that the sum equal 10, if you know that the 1st element equal 6?

Solution: $A = \{46, 55, 64\}$, $B = \{61, 62, 63, 64, 65, 66\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability (9/31)

Example3:

A dice is rolled twice. What is the probability that the sum equal 10, if you know that the 1st element equal 6?

Solution: $A = \{46, 55, 64\}$, $B = \{61, 62, 63, 64, 65, 66\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A) = 3/36$
$P(B) = 6/36$

$P(A \cap B) = 1/36$

$(A \cap B) = \{64\}$

Conditional Probability (9/31)

Example3:

A dice is rolled twice. What is the probability that the sum equal 10, if you know that the 1st element equal 6?

Solution: $A = \{46, 55, 64\}$, $B = \{61, 62, 63, 64, 65, 66\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{6/36} = 1/6$$

$P(A) = 3/36$
$P(B) = 6/36$

$P(A \cap B) = 1/36$

$(A \cap B) = \{64\}$

Conditional Probability (10/31)

Disjoint (or mutually exclusive):

S is a sample space, A, B are two events

$A, B \subseteq S$ and A, B **Disjoint** *or* **mutually exclusive**

$$\therefore P(A \cap B) = 0$$

$$\therefore P(A \cup B) = P(A) + P(B)$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$$

Conditional Probability (11/31)

Independence:

S is a sample space, A, B are two events
$A, B \subseteq S$ and A, B are **independent**

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) * P(B)}{P(B)} = P(A)$$

$$\therefore P(A \cap B) = P(A) * P(B)$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A) * P(B)$$

Conditional Probability (12/31)

Example4:

- If $P(A) = 0.2$, $P(B) = 0.3$ determine the following probabilities:

If A, and B are **disjoint** (mutually exclusive)

1) $P(A \cap B)$
2) $P(A \cup B)$
3) $P(A|B)$

Conditional Probability (13/31)

Example4:

- If $P(A) = 0.2$, $P(B) = 0.3$ determine the following probabilities:

If A, and B are **disjoint** (mutually exclusive)

Solution:

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) = 0.2 + 0.3 = 0.5$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$$

Conditional Probability (14/31)

Example5:

- If $P(A) = 0.2$, $P(B) = 0.3$ determine the following probabilities:

If $A,$ and B are **independent**

1) $P(A \cap B)$
2) $P(A \cup B)$
3) $P(A|B)$

Conditional Probability (15/31)

Example5:

- If $P(A) = 0.2$, $P(B) = 0.3$ determine the following probabilities:

If A, and B are **independent**

Solution:

$$P(A \cap B) = P(A) * P(B) = 0.2 * 03 = 0.06$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 - 0.06 = 0.44$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) * P(B)}{P(B)} = P(A) = 0.2$$

Conditional Probability (16/31)

Example6:

The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?

Conditional Probability (17/31)

Example6:

The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?

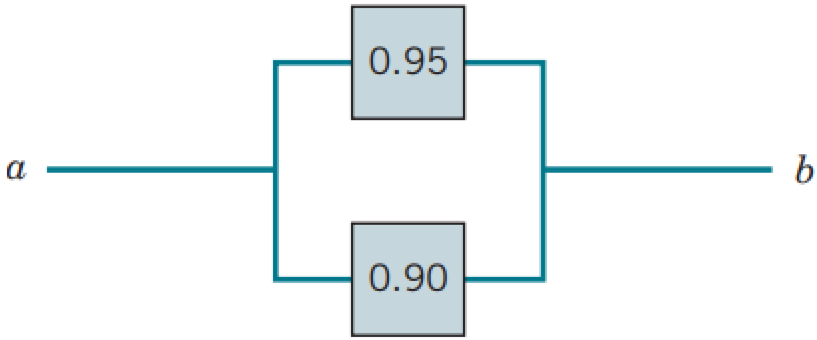
Let L and R denote the events that the left and right devices operate, respectively.

$$P(L \cap R) = P(L)P(R) = 0.80(0.90) = 0.72$$

Conditional Probability (18/31)

Example7:

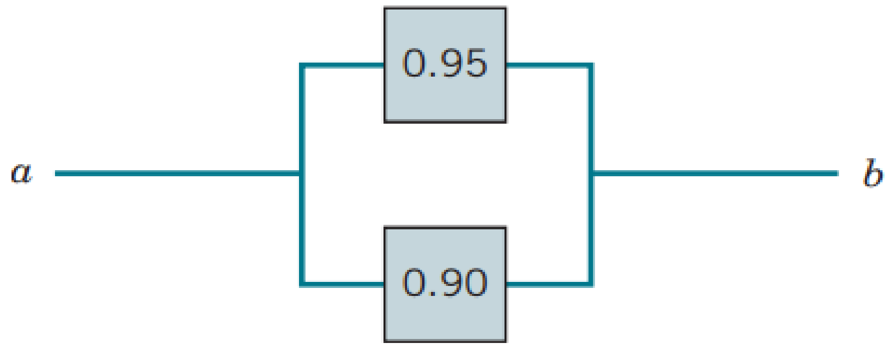
The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?



Conditional Probability (19/31)

Example7:

What is the probability that the circuit operates?



Let T and B denote the events that the top and bottom devices operate, respectively.

$$P(T \cup B) = P(T) + P(B) - P(T)P(B)$$

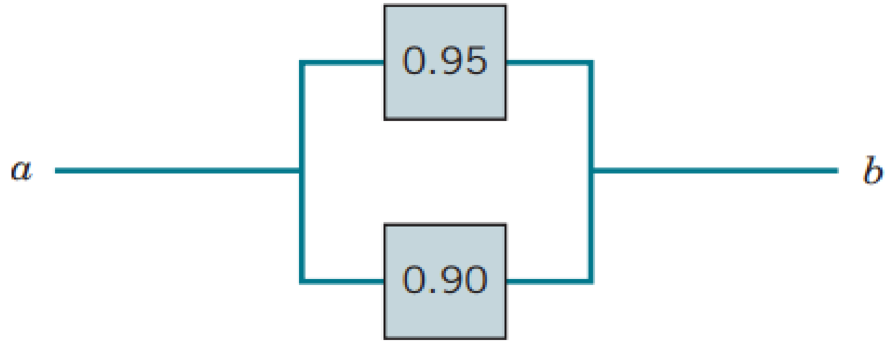
$$= 0.95 + 0.90 - (0.95)(0.90) = 0.995$$

Conditional Probability (20/31)

Example7:

Another Solution

What is the probability that the circuit operates?



$$P(T \cup B) = 1 - P(T \cup B)'$$

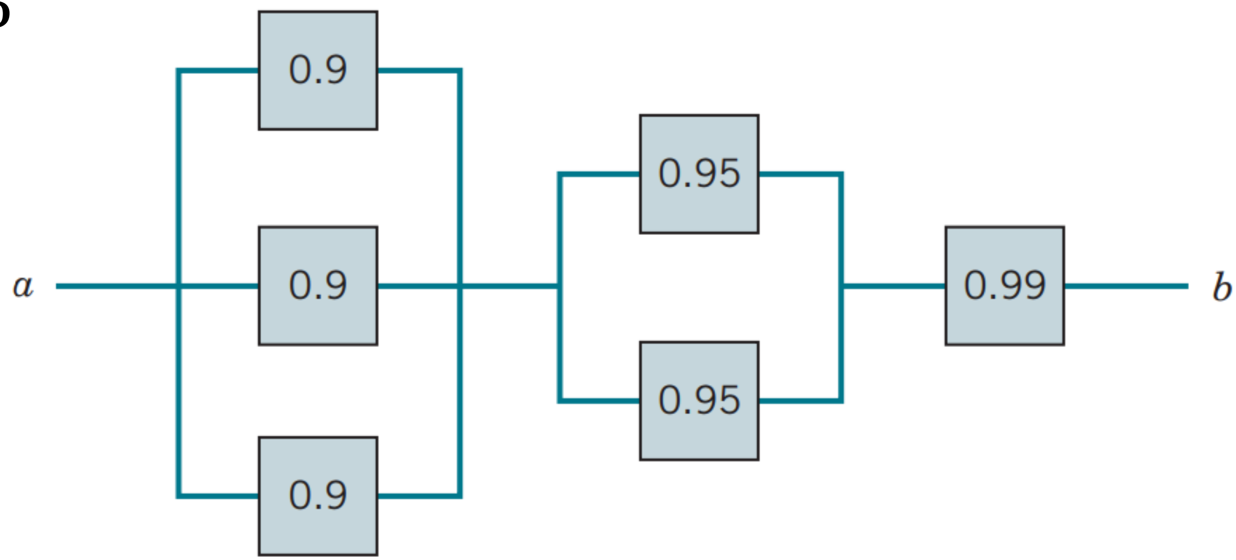
$$= 1 - P(T' \cap B') = 1 - P(T')P(B')$$

$$= 1 - (0.05)(0.10) = 0.995$$

Conditional Probability (21/31)

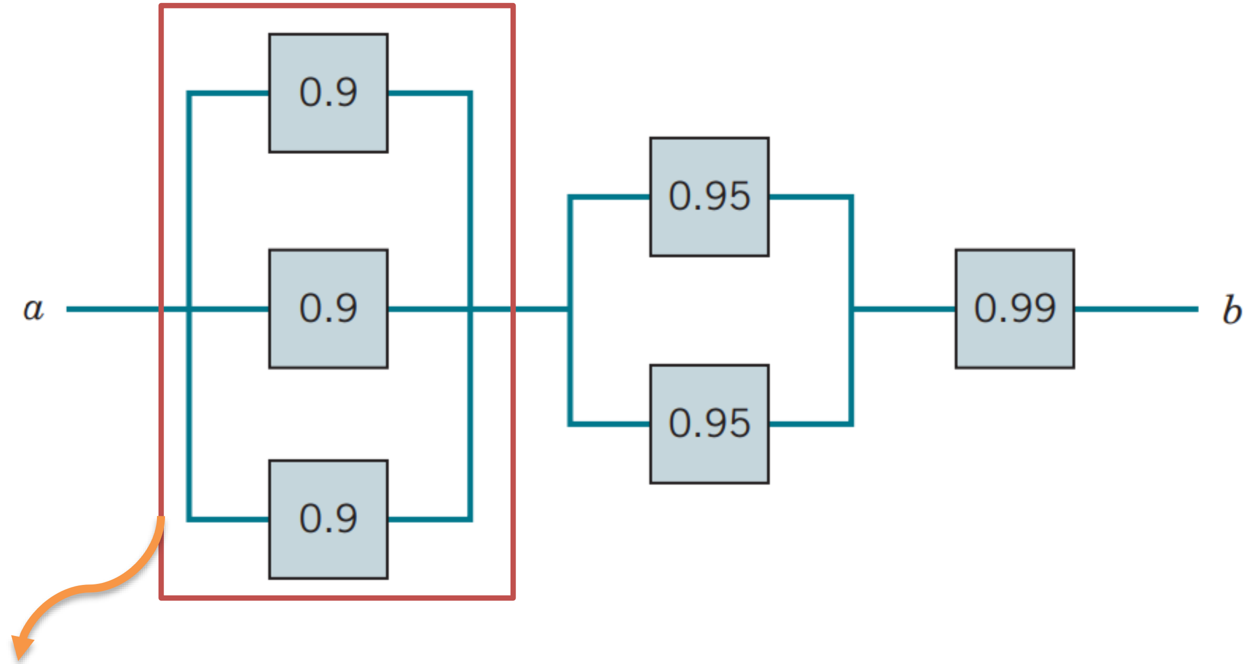
Example8:

The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?



Conditional Probability (22/31)

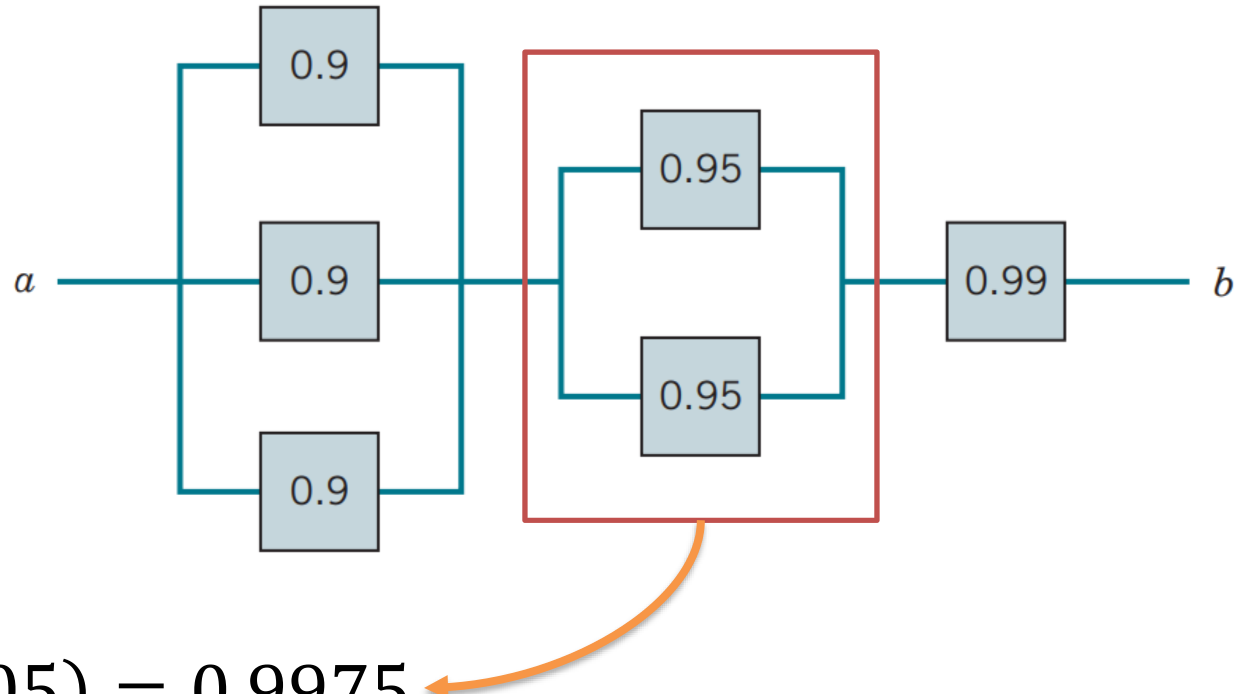
Example8:



$$= 1 - (0.10)(0.10)(0.10) = 0.999$$

Conditional Probability (23/31)

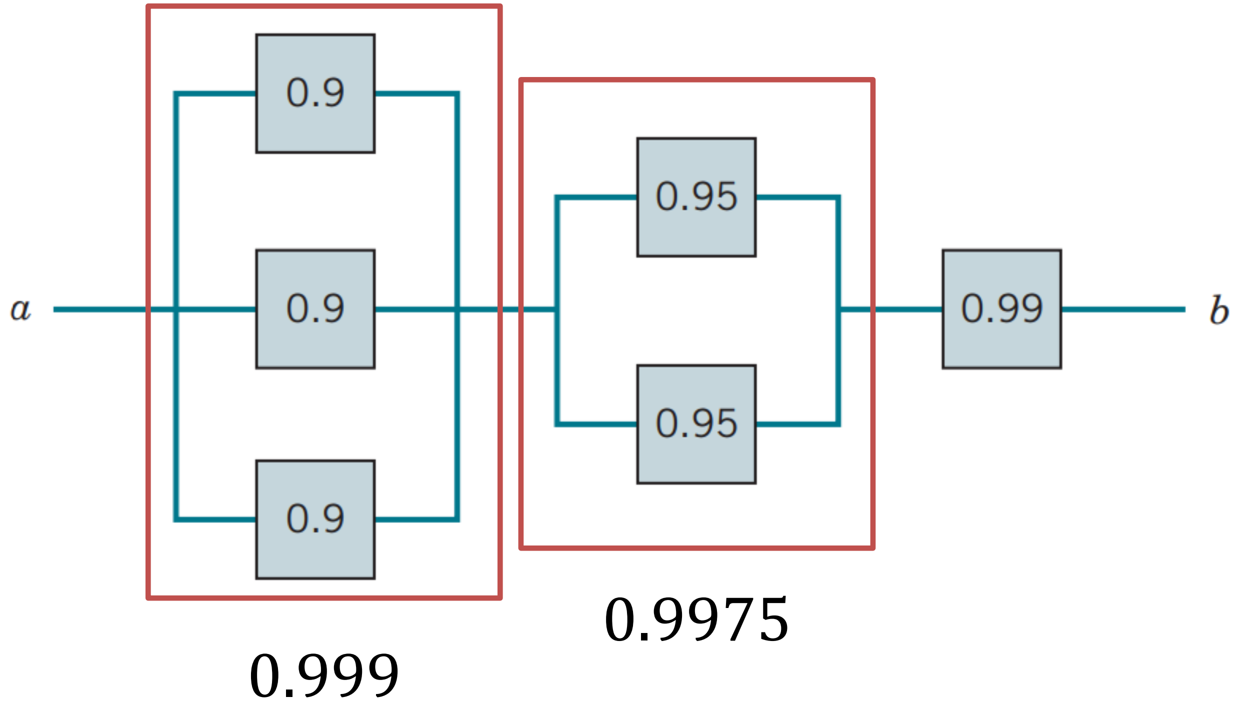
Example8:



$$= 1 - (0.05)(0.05) = 0.9975$$

Conditional Probability (24/31)

Example8:



$$= (0.999)(0.9975)(0.99) = 0.9865$$

Multiplication Rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad , \qquad for \;\; P(B) > 0$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad , \qquad for \;\; P(A) > 0$$

$$\boldsymbol{P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)}$$

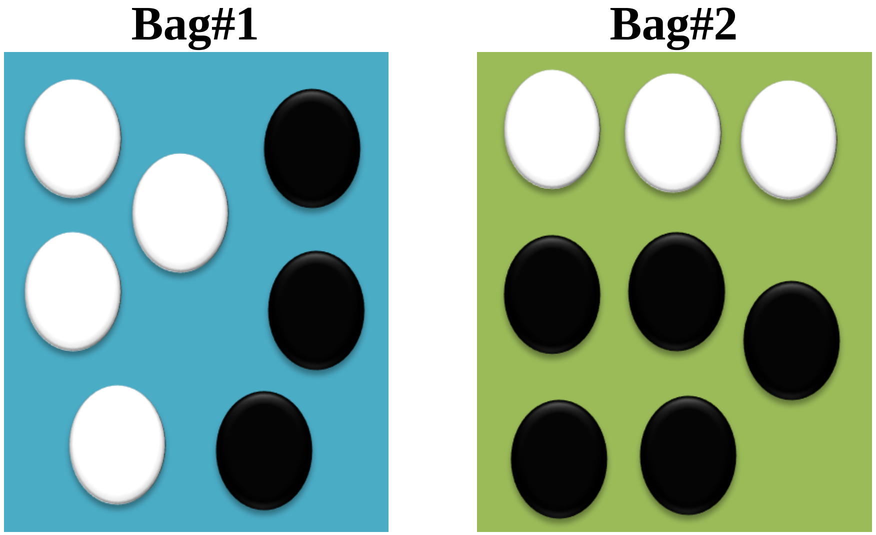
Conditional Probability (26/31)

Example9:

One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?

Example9:

One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?



Solution:

B_1: Black from bag#1

W_1: White from bag#1

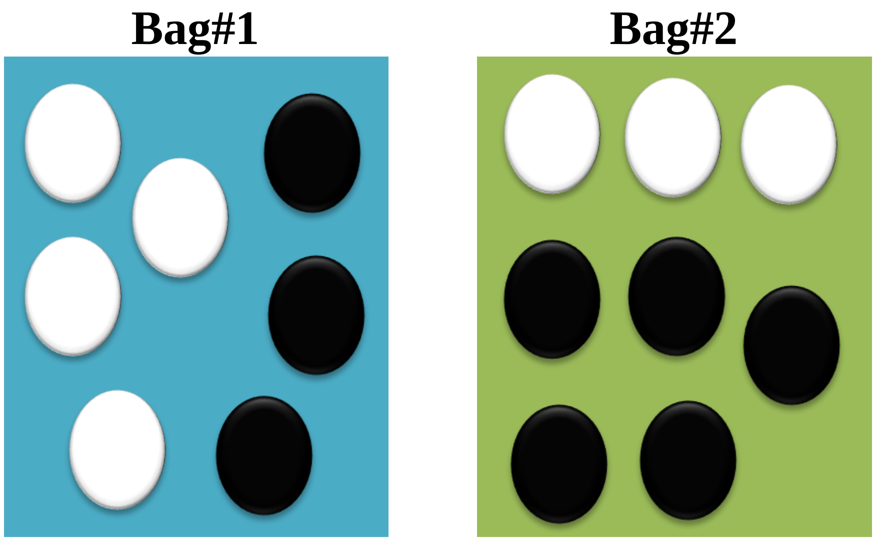
B_2: Black from bag#2

W_2: White from bag#2

Conditional Probability (27/31)

Example9: B_1 and B_2 or W_1 and B_2

One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?



Solution:

B_1: Black from bag#1

W_1: White from bag#1

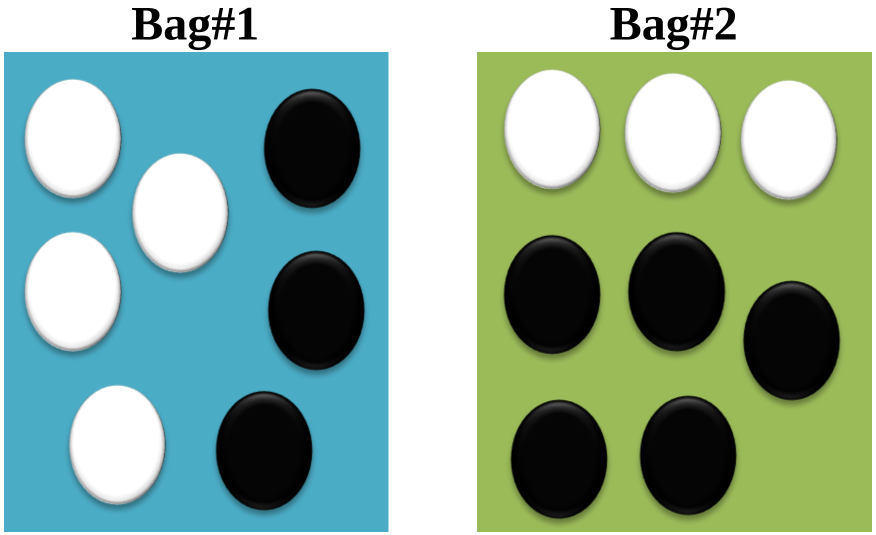
B_2: Black from bag#2

W_2: White from bag#2

Conditional Probability (28/31)

Example9: B_1 and B_2 or W_1 and B_2

$$P(B_1 \cap B_2) = P(B_2|B_1)P(B_1) = \left(\frac{6}{9}\right)\left(\frac{3}{7}\right)$$

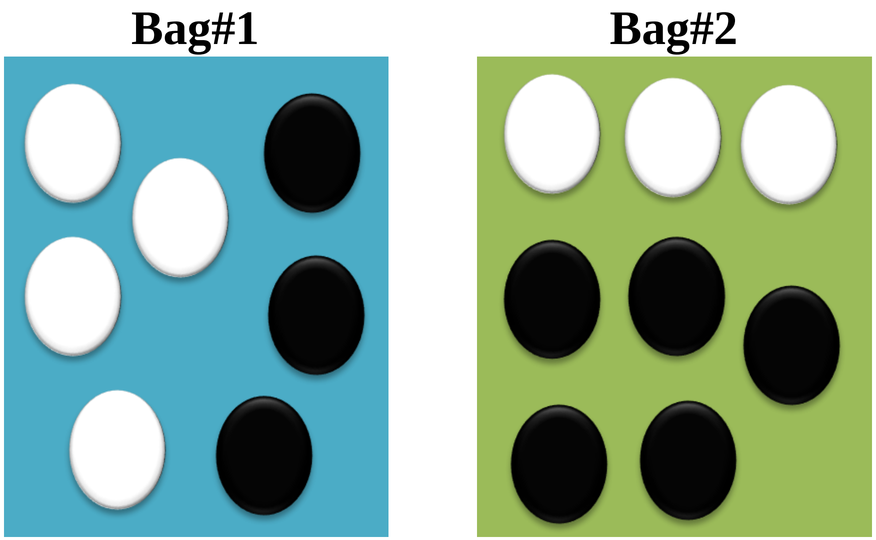


Conditional Probability (29/31)

Example9:

B_1 and B_2 or W_1 and B_2

$$P(W_1 \cap B_2) = P(B_2|W_1)P(W_1) = \left(\frac{5}{9}\right)\left(\frac{4}{7}\right)$$



Conditional Probability (30/31)

Example9: B_1 and B_2 or W_1 and B_2 **Disjoint**

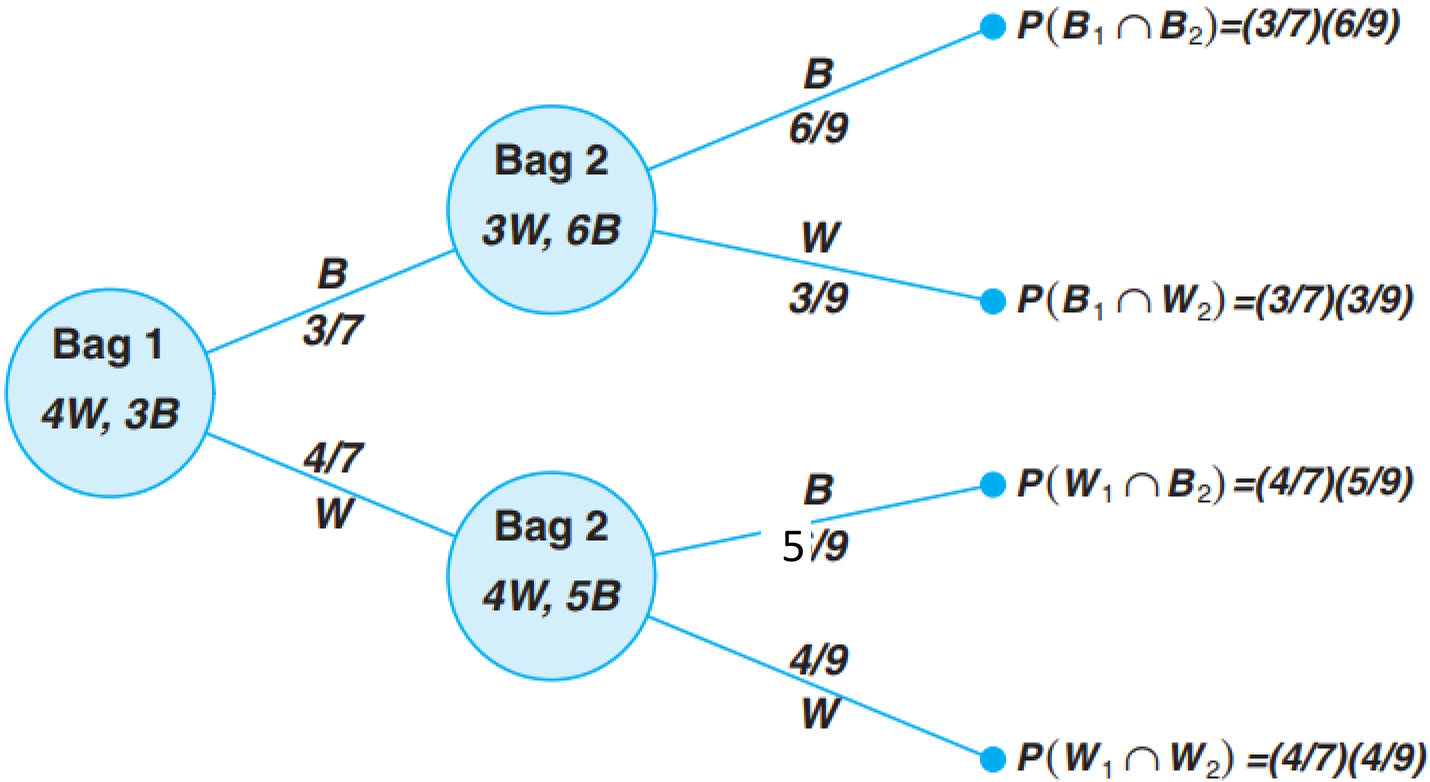
$$P(B_1 \cap B_2) = P(B_2|B_1)P(B_1) = \left(\frac{6}{9}\right)\left(\frac{3}{7}\right)$$

$$P(W_1 \cap B_2) = P(B_2|W_1)P(W_1) = \left(\frac{5}{9}\right)\left(\frac{4}{7}\right)$$

What is the probability that a ball now drawn from the second bag is black $= \left(\frac{6}{9}\right)\left(\frac{3}{7}\right) + \left(\frac{5}{9}\right)\left(\frac{4}{7}\right) = \frac{38}{63}$

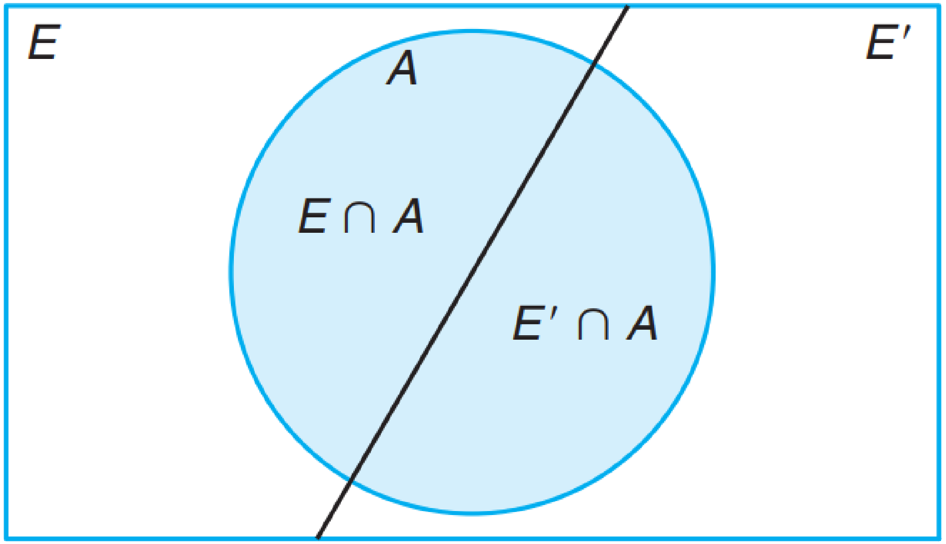
Conditional Probability (31/31)

Example9: B_1 and B_2 or W_1 and B_2 **Disjoint**



Total Probability Rule (1/10)

Total Probability Rule:



$$P(A) = P(E \cap A) \cup P(E' \cap A)$$
$$= P(E \cap A) + P(E' \cap A)$$
$$= P(A|E)P(E) + P(A|E')P(E')$$

Total Probability Rule (2/10)

Example10:

Consider the information about contamination in the following table.

$P(F|H) = 0.1$

$P(F|H') = 0.005$

$P(H) = 0.2$

$P(H') = 0.8$

$P(F)$?

Probability of Failure	Level of Contamination	Probability of Level
0.1	High	0.2
0.005	Not high	0.8

Let F denote the event that the product fails, and let H denote the event that the chip is exposed to high levels of contamination.

Total Probability Rule (3/10)

Example10:

Consider the information about contamination in the following table.

$P(F|H) = 0.1$

$P(F|H') = 0.005$

$P(H) = 0.2$

$P(H') = 0.8$

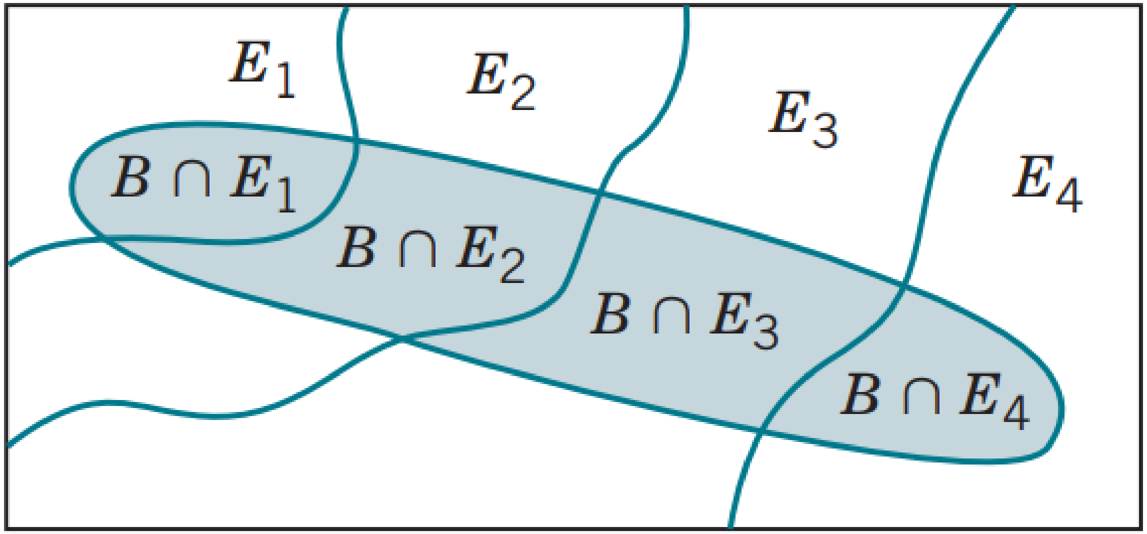
$P(F)$?

Probability of Failure	Level of Contamination	Probability of Level
0.1	High	0.2
0.005	Not high	0.8

$$P(F) = P(F|H)P(H) + P(F|H')P(H')$$
$$= (0.1)(0.2) + (0.005)(0.8) = 0.024$$

Total Probability Rule (4/10)

Total Probability Rule (Multiple Events):



$$B = (B \cap E_1) \cup (B \cap E_2) \cup (B \cap E_3) \cup (B \cap E_4)$$

$$P(B) = P(B \cap E_1) + P(B \cap E_2) + P(B \cap E_3) + P(B \cap E_4)$$

Total Probability Rule (5/10)

Example11:

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

Total Probability Rule (5/10)

Example11:

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

***D*: the product is defective. Find $P(D)$?**

Total Probability Rule (6/10)

Example11: $P(B_1) = 0.3, P(B_2) = 0.45, P(B_3) = 0.25$

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

$$P(D|B_1) = 0.02,$$
$$P(D|B_2) = 0.03,$$
$$P(D|B_3) = 0.02.$$

Total Probability Rule (7/10)

Example11: $P(B_1) = 0.3, P(B_2) = 0.45, P(B_3) = 0.25$

$P(D|B_1) = 0.02,$
$P(D|B_2) = 0.03,$
$P(D|B_3) = 0.02.$

Applying the total probability rule , we can write

$$P(D)$$
$$= P(D|B_1)P(B_1) + P(D|B_2)P(B_2) + P(D|B_3)P(B_3)$$
$$= 0.02(0.3) + 0.03(0.45) + 0.02(0.25) = 0.0245$$

Total Probability Rule (8/10)

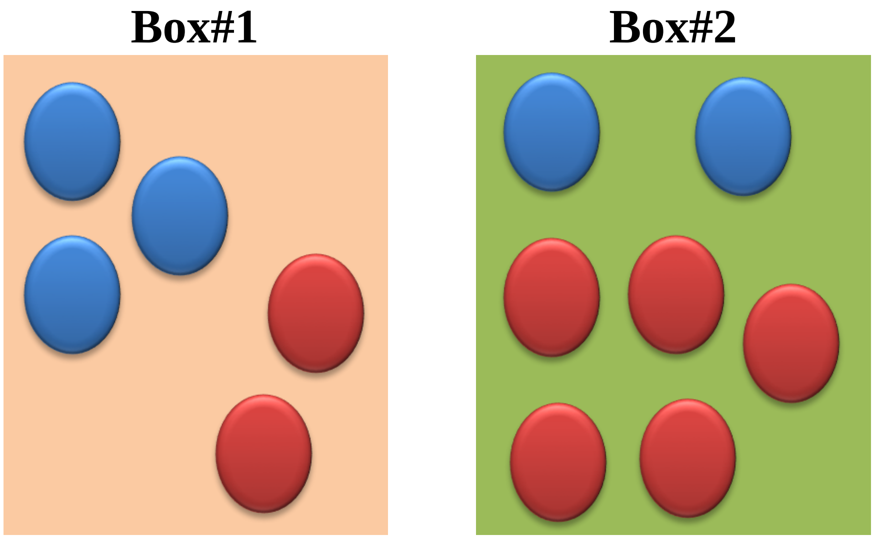
Example12:

Box#1 contains 2 red balls and 3 blue balls; Box#2 contains 5 red balls and 2 blue balls. If the selection of two boxes is equally likely, and you selected one ball, what is the probability that it is red?

Total Probability Rule (8/10)

Example12:

Box#1 contains 2 red balls and 3 blue balls; Box#2 contains 5 red balls and 2 blue balls. If the selection of two boxes is equally likely, and you selected one ball, what is the probability that it is red?

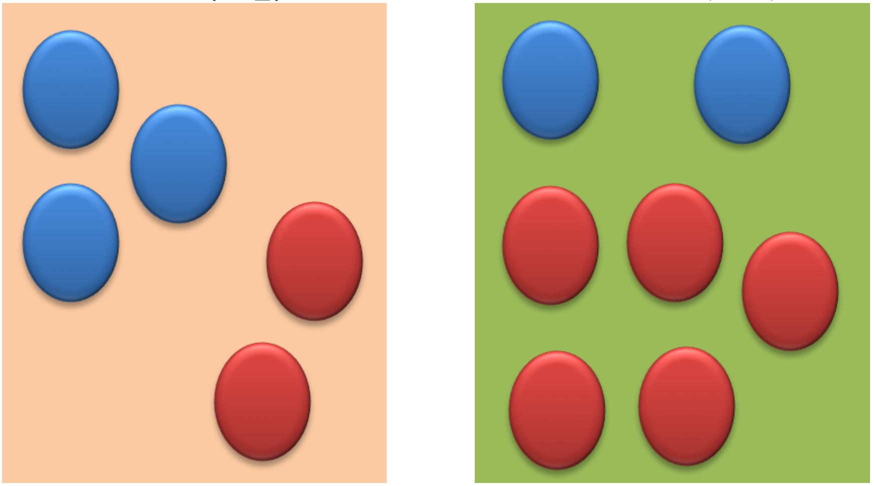


Total Probability Rule (9/10)

Example12:

Box#1 contains 2 red balls and 3 blue balls; Box#2 contains 5 red balls and 2 blue balls**. If the selection of two boxes is equally likely**, and you selected one ball, what is the probability that it is red?

Box#1 (B_1) **Box#2 (B_2)**



$P(B_1) = P(B_2) = 0.5$

$R: read, \qquad B: blue$

Find $P(R)$?

Total Probability Rule (10/10)

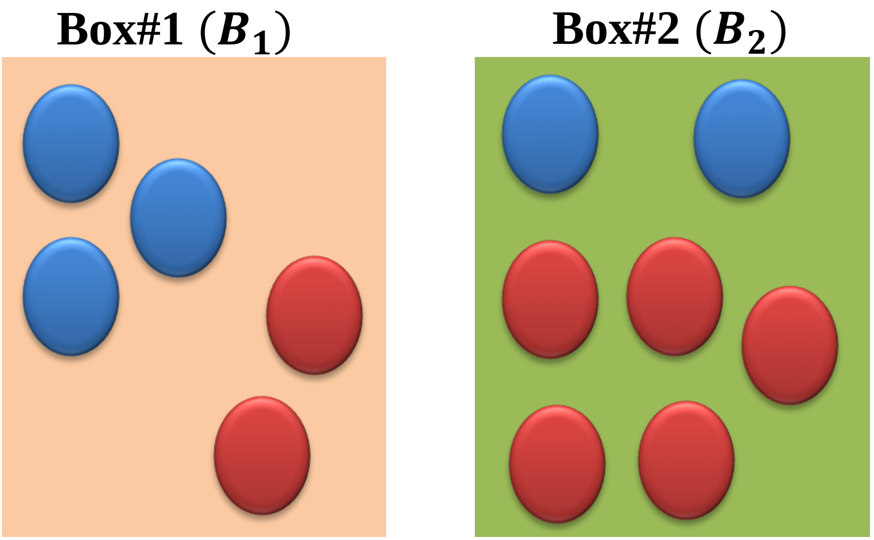
Example12:

$P(B_1) = P(B_2) = 0.5$

$R: read, \qquad B: blue$

$P(R|B_1) = {}^{2}/_{5} = 0.4$

$P(R|B_2) = {}^{5}/_{7} = 0.7143$



$$P(R) = P(R|B_1)P(B_1) + P(R|B_2)P(B_2)$$

$$= (0.4)(0.5) + (0.7143)(0.5) = 0.55715$$

Bayes' Rule (1/11)

From the definition of conditional probability,

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

Now, considering the second and last terms in the preceding expression, we can write

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} \quad \text{for} \quad P(B) > 0$$

Bayes' Rule (2/11)

If $E_1, E_2, \ldots, E_k$ are k mutually exclusive and exhaustive events and B is any event,

$$P(E_1 \mid B) = \frac{P(B \mid E_1)P(E_1)}{P(B \mid E_1)P(E_1) + P(B \mid E_2)P(E_2) + \cdots + P(B \mid E_k)P(E_k)}$$

for $P(B) > 0$

Bayes’ Rule (3/11)

Example1:

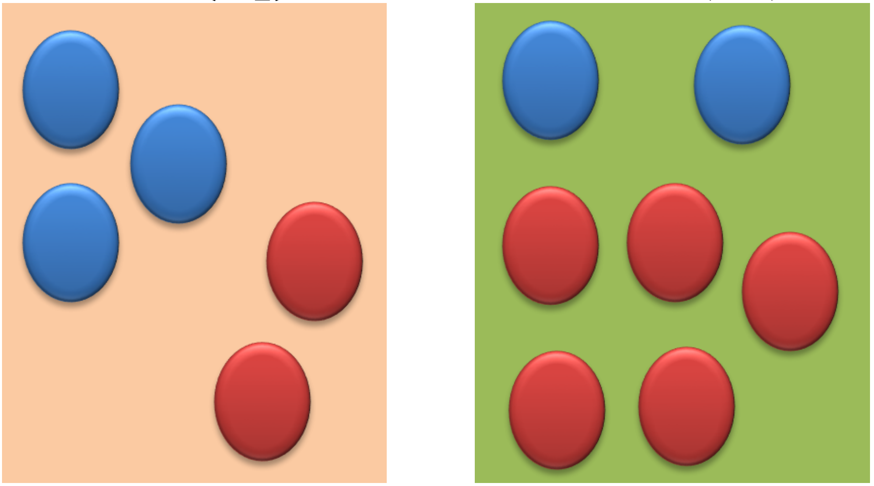
Box#1 contains 2 red balls and 3 blue balls; Box#2 contains 5 red balls and 2 blue balls. If the selection of two boxes is equally likely, and the selected ball was red, what is the probability that it is from Box#1?

$P(B_1) = P(B_2) = 0.5$

$R: read, \qquad B: blue$

Find $\boldsymbol{P(B_1|R)}$ **?**

Box#1 ($\boldsymbol{B_1}$) **Box#2 ($\boldsymbol{B_2}$)**



Bayes' Rule (4/11)

Example1:

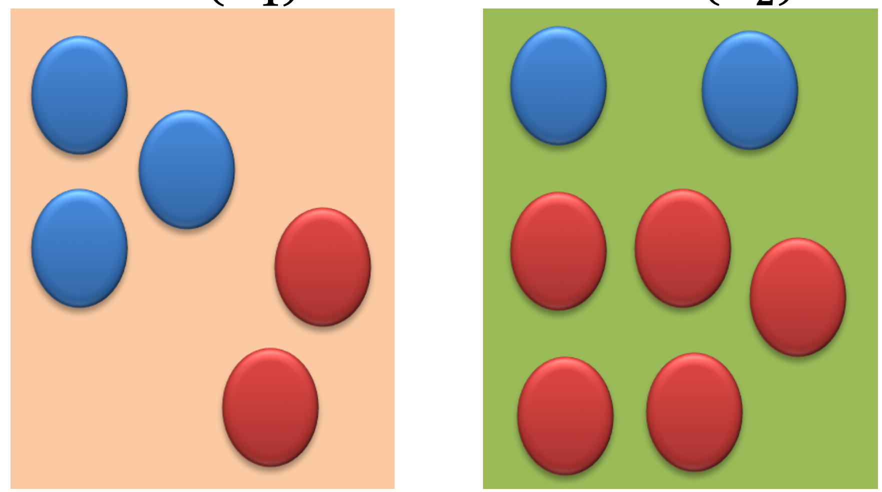
$$P(B_1) = P(B_2) = 0.5$$

$R: read, \qquad B: blue$

$$P(R|B_1) = {}^{2}/_{5} = 0.4$$

$$P(R|B_2) = {}^{5}/_{7} = 0.7143$$

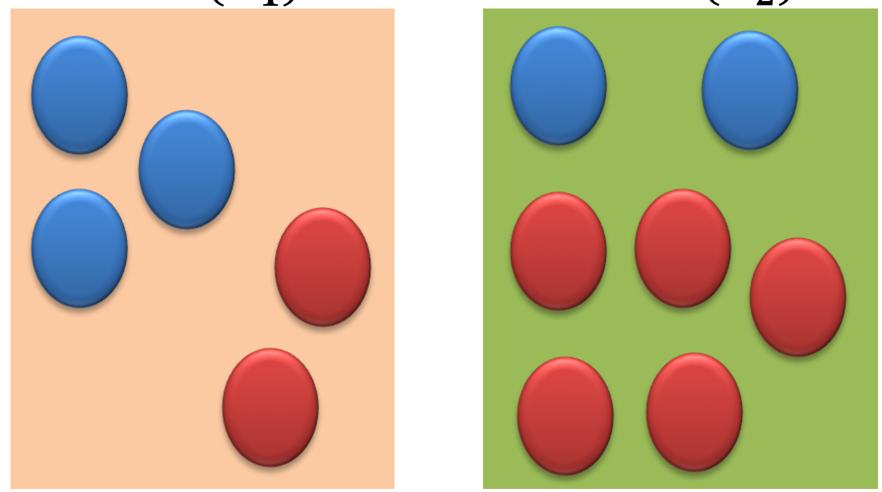
Box#1 (B_1) **Box#2** (B_2)



Bayes' Rule (5/11)

Example1:

Box#1 $(\boldsymbol{B_1})$ Box#2 $(\boldsymbol{B_2})$



$$P(B_1) = P(B_2) = 0.5$$

$$R: read, \qquad B: blue$$

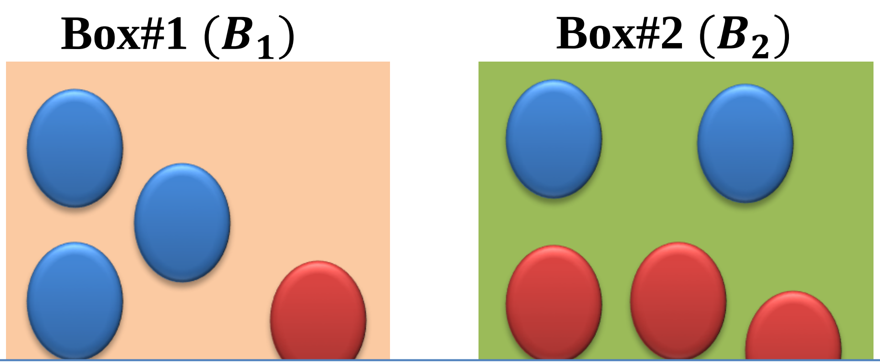
$$P(R|B_1) = {}^{2}/_{5} = 0.4$$

$$P(R|B_2) = {}^{5}/_{7} = 0.7143$$

$$P(B_1|R) = \frac{P(R|B_1)P(B_1)}{P(R)} = \frac{(0.4)(0.5)}{P(R)}$$

Bayes' Rule (6/11)

Example1:



$$P(B_1) = P(B_2) = 0.5$$

$R: read, \qquad B: blue$

$$P(R|B_1) = {}^{2}/_{5}$$

$$P(R|B_2) = {}^{5}/_{7}$$

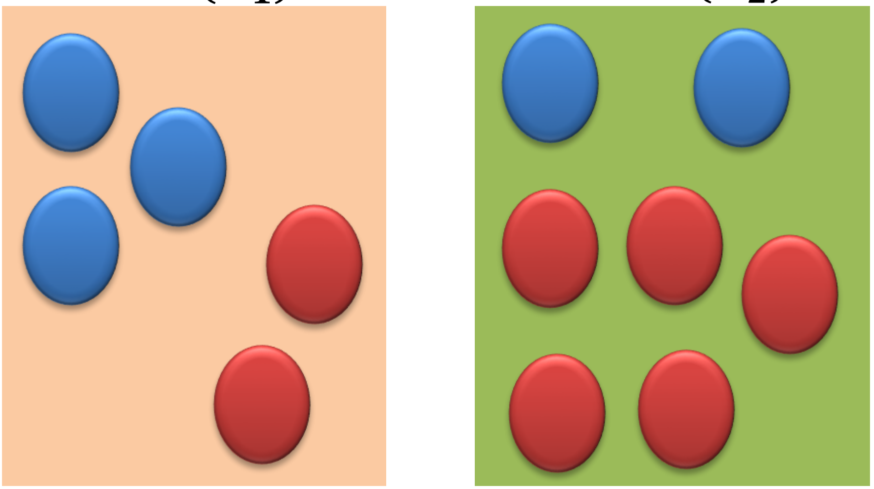
$$P(R) = P(R|B_1)P(B_1) + P(R|B_2)P(B_2)$$
$$= (0.4)(0.5) + (0.7143)(0.5) = 0.55715$$

$$P(B_1|R) = \frac{P(R|B_1)P(B_1)}{P(R)} = \frac{(0.4)(0.5)}{P(R)}$$

Bayes’ Rule (7/11)

Example1:

Box#1 $(\boldsymbol{B_1})$ **Box#2** $(\boldsymbol{B_2})$



$$P(B_1) = P(B_2) = 0.5$$

$R: read, \qquad B: blue$

$$P(R|B_1) = {}^{2}/_{5} = 0.4$$

$$P(R|B_2) = {}^{5}/_{7} = 0.7143$$

$$P(B_1|R) = \frac{P(R|B_1)P(B_1)}{P(R)} = \frac{(0.4)(0.5)}{P(R)} = \frac{0.2}{0.55715} = 0.35897$$

Bayes' Rule (8/11)

Example2:

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. If a product was chosen randomly and found to be defective, what is the probability that it was made by machine B_3?

D: the product is defective. Find $P(B_3|D)$?

Bayes' Rule (8/11)

Example2: $P(B_1) = 0.3, P(B_2) = 0.45, P(B_3) = 0.25$

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. If a product was chosen randomly and found to be defective, what is the probability that it was made by machine B_3?

$$P(D|B_1) = 0.02,$$
$$P(D|B_2) = 0.03,$$
$$P(D|B_3) = 0.02.$$

Bayes' Rule (9/11)

Example2: $P(B_1) = 0.3, P(B_2) = 0.45, P(B_3) = 0.25$

$P(D|B_1) = 0.02,$
$P(D|B_2) = 0.03,$
$P(D|B_3) = 0.02.$

Using Bayes' rule to write

$$P(B_3|D) = \frac{P(D|B_3)P(B_3)}{P(D)}$$

Bayes' Rule (10/11)

Example2:

Applying the total probability rule , we can write

$$P(D)$$
$$= P(D|B_1)P(B_1) + P(D|B_2)P(B_2) + P(D|B_3)P(B_3)$$
$$= 0.02(0.3) + 0.03(0.45) + 0.02(0.25) = 0.0245$$

Using Bayes' rule to write

$$P(B_3|D) = \frac{P(D|B_3)P(B_3)}{P(D)}$$

Bayes' Rule (11/11)

Example2: $P(B_1) = 0.3, P(B_2) = 0.45, P(B_3) = 0.25$

$P(D|B_1) = 0.02,$
$P(D|B_2) = 0.03,$
$P(D|B_3) = 0.02.$

Using Bayes' rule to write

$$P(B_3|D) = \frac{P(D|B_3)P(B_3)}{0.0245} = \frac{(0.02)(0.25)}{0.0245} = 0.2041$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-

Lecture #3: https://www.youtube.com/watch?v=raeVQxzY7iE&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=3

https://www.youtube.com/watch?v=zWDzNUTfk9s&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=4 Up to time 00:41:39

Thank You

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